Exam. Code : 211003
Subject Code : 3854

M.Sc. Mathematics 3rd Semester

ADVANCED NUMERICAL ANALYSIS

Paper—MATH-579

Time Allowed—Three Hours] [Maximum Marks—100

Note :—Attempt any **TWO** questions from each Unit. All questions carry equal marks.

UNIT—I

1. Construct a piecewise cubic spline interpolant for the curve passing through (1, 2), (2, 3), (3, 5) with natural boundary conditions.

2. Calculate the Fourier integral numerically

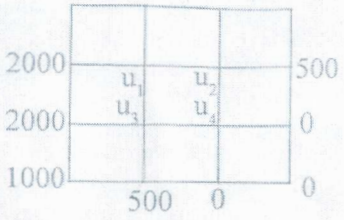
$$\int_0^{\infty} e^{-t/6} \sin \omega t \, dt .$$

3. Using economization of series, express e^x in terms of Chebyshev series upto $n = 5$.

4. Is $s(x) = a_0 + a_1 \sin t + a_2 \sin 2t + a_3 \sin 3t + a_4 \sin 4t$ a spline function ? Explain.

UNIT—II

5. Values of u(x, y) on the boundary of square are given. Evaluate the function u(x, y) satisfying Laplace equation $\nabla^2 u = 0$ at pivotal points by using Jacobi's method.



6. From the BVP $u_t = u_{xx}$, u(x, 0) = 20, u(0, t) = 0, u(5, t) = 100, $0 < x < 5$, $t \geq 0$, compute the value of u for h = 0.2 and $\alpha = 0.02$ using finite difference method.

7. Solve the hyperbolic PDE $u_{tt} = u_{xx}$, subject to the conditions that :

   u(0, t) = 0, u(1, t) = 0, u_t(x, 0) = 0, u(x, 0) = 10 + x(1 – x).

8. Solve $u_{xx} + u_{yy} = 0$, by SOR method subject to conditions that :

   u(x, 0) = u(0, y) = 0, u(1, y) = y – y^3, u(x, 1) = x^3 – x.

UNIT—III

9. Using Galerkin method, solve $u'' + (1 + x^2)u + 1 = 0$, $u(\pm 1) = 0$.

10. Using Rayleigh Ritz method, solve the BVP $-u'' + u = x$, subject to u'(0) = 1, u'(1) = 2.

11. Using shooting method, solve the BVP $u'' + u = x$ subject to $u(0) = 0$, $u(1) = 0$.

12. Solve the BVP $u_t = u_{xx} - 100x$, for $0 \le x < 1$, $t > 0$, given that $u(x, 0) = u(0, t) = u(1, t) = 0$, using finite element method.

UNIT—IV

13. Define random numbers. Discuss different type of random numbers generators.

14. Estimate value of π using Buffon's needle problem.

15. Apply inverse transform method to generate random variables from the extreme value distribution which has cdf $F(x) = 1 - e^{-\exp\left(\frac{x-\mu}{\sigma}\right)}, -\infty < x < \infty, (\sigma > 0)$.

16. Let a random variable X have pdf

$$f(x) = \begin{cases} \frac{1}{4}, & 0 < x < 1 \\ x - \frac{3}{4}, & 1 \le x \le 2 \end{cases}$$

, generate a random variable from f(x) using acceptance rejection method using the proposal density $g(x) = \frac{1}{2}$, $0 \le x \le 2$.

UNIT—V

17. Cans from a Coke production line follow the following distribution regarding their weights.

Weight (in oz)	11.8 or less	11.9	12.0
Percentage (%)	10	20	40
Weight (in oz)	12.1	12.2 or more	
Percentage (%)	20	10	

Using Monte Carlo method simulate the process and find what percentage of cans are at or above 12 oz.

18. Classify different simulation models.
19. What is importance sampling technique ? Give its algorithm.
20. Discuss variance reduction technique of Monte Carlo simulation.